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RESEARCH OF RESISTANCE OF MOVEMENT OF A FOUR-AXLE WAGON WITH VARIOUS WHEELS DESIGNS

VÝSKUM JAZDNÉHO ODPORU ŠTVRONÁPRAVOVÉHO VAGÓNA S RÔZNYMI KONŠTRUKCIAMI KOLIES

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Abstract

The article contains description of a mathematical model of the movement of a four-axle freight wagon on a rail track. With the help of mathematical modelling, studies of the running resistance of a wagon with wheels of various design schemes have been carried out. There are compared two wheel designs, namely the design of a traditional wheel with a monolithic production tread surface and the flange and the perspective wheel design, which allows independent rotating of support surface of the wheel and of its guiding surface, i.e. of its flange. Potential advantages of using the wheels of a perspective design scheme in a bogie of a wagon are revealed, which consist in reducing its resistance of movement.

Keywords

wagon, wheel, flange, mathematical modelling, resistance of movement

Abstrakt

Príspevok obsahuje opis matematického modelu pohybu štvornápravového nákladného vagóna na železničnej trati. Prostredníctvom matematického modelovania boli vykonané štúdie jazdného odporu vagóna s kolesami s rôznym konštrukčným usporiadaním. Porovnávané sú dve konštrukcie kolies, konkrétne konštrukcia tradičného kolesa, ktorého jazdná plocha a okolesník tvoria jeden celok a perspektívna konštrukcia kolesa, ktorá umožňuje nezávislú rotáciu jazdnej plochy kolesa a vodiacej plochy, t. j. okolesníka. Predstavené sú potenciálne výhody použitia kolies s perspektívnou konštrukciou v podvozku vagóna, ktoré tkvejú v redukovaní jazdného odporu.

Kľúčové slová

vagón, koleso, okolesník, matematické modelovanie, jazdný odpor

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1 INTRODUCTION

One of the ways to reduce the resistance to the movement of rail vehicles can be the use of wheels of a perspective design scheme (PDS) in their bogies. This design allows to rotate a flange and a tread surface of a railway wheel independently to each other about their common axis of rotation. The results of studies [1–5] have shown the peculiarities of movement on the rail of such wheels and the possibility to reduce the slip speed in the flange contact. This, in turn, leads to reducing of the resistance to movement of a wheel and wear in the contact surfaces. In this regard, it seems to be appropriate to further study the prospects for reducing resistance to movement of rail vehicles by means of the use of PDS wheels by mathematical modelling of the movement of an entire bogie or even a rail vehicle [6, 7].

2 ANALYSIS OF THE CURRENT STATE

A review of the scientific literature in this research area has shown that at present, a significant number of models, which are created within various degrees of a system idealization and the description of details have been proposed including the mathematical description of movement of rail vehicles. These approaches have been derived for various given tasks. This article presents a choice of the vehicle design scheme and a mathematical modelling of movement, which has been carried out based on the analysis of works [2, 8–10], and taking into account the peculiarities of the interaction of the PDS wheel with the rail [1–3].

The main purpose of this work is a mathematical description of the movement of a four-axle freight car with wheels of various design schemes to study their influence on the resistance to movement along a rail track.

3 METHODS OF A PROBLEM SOLUTION

The research comes from a mathematical model of the vehicle movement, which is presented in the works of scientists from the Dnepropetrovsk National University of Railway Transport named by V. A. Lazaryan [11–15].

A vehicle model was considered as a system including 19 rigid bodies, namely a body, two bolsters, four side frames of bogies, four wheelsets and eight flanges (having the possibility of independent rotation relative to the wheels around their common axis) with rigid and visco-elastic connections between them. A scheme of the considered model is shown in Fig. 1.

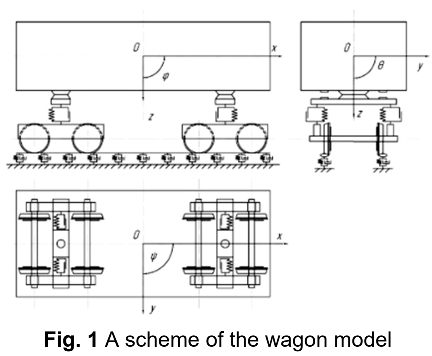

Fig. 1 A scheme of the wagon model

The description of the wagon movement, i.e. its oscillations, is within a coordinate system, that *x* axis is for a longitudinal movement, *y* axis is the lateral axis and *z* axis heading downwards as it is standard for rail vehicles. It was assumed, that the wagon moves at the constant speed v and the track is straight with curves.

The coordinates of the centres of gravity of individual bodies are designated by letters "*x*", "*y*" and "*z*". The superscript "*a*" stands for the car body, "*b*" for the bolster, "*s*" for the side frame of the bogie, "*w*" for the wheelset, "*g*" for the flange, "*r*" for the rail. The lower index "*i*" designates the number of the bogie in the direction of travel, the lower index "*j*" designates one of the sides of the carriage (1 – left side, 2 – right), the letter "m" is the number of the wheelset in the bogie (1 – the first in the direction of travel, 2 – the second). Angular displacements relative to the axes O_x, O_y and O_z are denoted by symbols "ζ", "φ" and "ψ" and masses and moments of inertia are m, I_x, I_y and I_z.

The components of forces and moments are denoted by the corresponding capital letters, the subscripts for which have the same meaning as the subscripts for coordinates. The upper ones are the designation of the body from which the force (moment) acts, and the designation of the body, separated by a comma, and the designation of the body, on which it acts.

3.1 Equations of motion

The car body has six degrees of freedom (movement and turns relative to three mutually perpendicular axes), and it is affected by longitudinal forces in automatic couplers X_i, longitudinal $X_{ij}^{b(sk),a}$ and vertical $Z_{ij}^{b(sk),a}$ forces in sideways, forces $X_{ij}^{b(c),a}$, $Y_{ij}^{b(c),a}$, $Z_{ij}^{b(c),a}$ and moments $\Theta_{ij}^{b(c),a}$, $\Psi_{ij}^{b(c),a}$ in centre plate nodes. The movement of the car body is described by the following differential equations:

$$\begin{aligned}
m^a \cdot \ddot{x}^a &= \sum_{i=1}^{2}\left(X_i + X_i^{b(c),a} + \sum_{i=1}^{2} X_{ij}^{b(sk),a} \right)\\
m^a \cdot \ddot{y}^a &= \sum_{i=1}^{2} Y_i^{b(c),a} - m^a \cdot a_0\\
m^a \cdot \ddot{z}^a &= \sum_{i=1}^{2}\left(Z_i^{b(c),a} + \sum_{i=1}^{2} Z_{ij}^{b(sk),a} \right) + m^a \cdot g\\
I_x^a \cdot \ddot{\vartheta}^a &= \sum_{i=1}^{2}\left(-h \cdot Y_i^{b(c),a} + b_2 \cdot \sum_{i=1}^{2} Z_{ij}^{b(sk),a} + \Theta_i^{b(c),a} \right)\\
I_y^a \cdot \ddot{\varphi}^a &= \sum_{i=1}^{2}\left[h \cdot \left(X_i + X_i^{b(c),a} + \sum_{i=1}^{2} X_{ij}^{b(sk),a} \right) + \left(-1\right)^j \cdot l \cdot \left(Z_{ij}^{b(sk),a} + \sum_{i=1}^{2} Z_{ij}^{b(sk),a} \right) \right]\\
I_z^a \cdot \ddot{\psi}^a &= \sum_{i=1}^{2}\left(-b_2 \cdot \sum_{i=1}^{2} \left(-1\right)^j \cdot X_{ij}^{b(sk),a} - \left(-1\right)^j \cdot l \cdot Y_i^{b(c),a} + \Psi_i^{b(c),a} \right)
\end{aligned}, \qquad (1)$$

where a_0 is exceptional acceleration, h is a distance from the centre of gravity of the body to the plane of support of the body on the bogies, $2l$ is a wagon base, $2 \cdot b_2$ is a distance between sideways and g is gravitational acceleration.

Each bolster has five degrees of freedom (rotation about the O_y axis is not considered). In addition to the corresponding forces from the side of the car body, forces act on it $X_{ij}^{s,b}$, $Y_{ij}^{s,b}$, $Z_{ij}^{s,b}$ from the side of bogies frames:

$$\begin{aligned}
m^b \cdot \ddot{x}^b &= X_i^{a(c),b} + \sum_{i=1}^{2}\left(X_{ij}^{a(sk),b} + X_{ij}^{s,b} \right)\\
m^b \cdot \ddot{y}^b &= Y_i^{a(c),b} + \sum_{i=1}^{2} Y_{ij}^{s,b} - m^b \cdot a_0\\
m^b \cdot \ddot{z}^b &= Z_i^{a(c),b} + \sum_{i=1}^{2}\left(Z_{ij}^{a(sk),b} + Z_{ij}^{s,b} \right) + m^b \cdot g\\
I_x^b \cdot \ddot{\vartheta}_i^b &= \sum_{i=1}^{2} \left(-1\right)^j \cdot \left(b_2 \cdot Z_{ij}^{a(sk),b} + b_1 \cdot Z_{ij}^{s,b} \right) + \Theta_i^{a(c),b}\\
I_z^a \cdot \ddot{\psi}_i^b &= \sum_{i=1}^{2} \left(-1\right)^j \cdot \left(b_2 \cdot X_{ij}^{a(sk),b} + b_1 \cdot X_{ij}^{s,b} \right) + \Psi_i^{a(c),b}
\end{aligned}, \qquad (2)$$

where $2 \cdot b_1$ is a lateral distance between springs of the suspension system of one bogie.

Each side frame has five degrees of freedom, namely x_{ij}^{s}, y_{ij}^{s}, z_{ij}^{s}, φ_{ij}^{s}, ψ_{ij}^{s} and the forces arising in the spring suspension and in the axle boxes act on it $X_{imj}^{w,s}$, $Y_{imj}^{w,s}$, $Z_{imj}^{w,s}$ and also moments $\Psi_{imj}^{w,s}$ attached in axle boxes:

$$\begin{aligned} m^{s}\cdot\ddot{x}_{ij}^{s} &= X_{ij}^{b,s}+\sum_{m=1}^{2}X_{imj}^{w,s} \\ m^{s}\cdot\ddot{y}_{ij}^{s} &= Y_{ij}^{b,s}+\sum_{m=1}^{2}Y_{imj}^{w,s}-m^{s}\cdot a_{0} \\ m^{s}\cdot\ddot{z}_{ij}^{s} &= Z_{ij}^{b,s}+\sum_{m=1}^{2}Z_{imj}^{w,s}+m^{s}\cdot g \\ I_{y}^{s}\cdot\ddot{\varphi}_{ij}^{s} &= l_{1}\cdot\sum_{m=1}^{2}\left(-1\right)^{m}\cdot Z_{imj}^{w,s} \\ I_{z}^{s}\cdot\ddot{\psi}_{ij}^{s} &= \sum_{m=1}^{2}\left[-l_{1}\cdot\left(-1\right)^{m}\cdot Y_{imj}^{w,s}+\Psi_{imj}^{w,s}\right] \end{aligned}, \tag{3}$$

where $2\cdot l_1$ is a bogie base.

Every wheelset has six degrees of freedom. From the side of the rails, forces act on it $X_{imj}^{r,w}$, $Y_{imj}^{r,w}$, $Z_{imj}^{r,w}$ and moments $\Theta_{imj}^{r,w}$, $\Phi_{imj}^{r,w}$, $\Psi_{imj}^{r,w}$ (which are the projections of the turning creep moment).

From the side of the guide surfaces of the wheels, moments $\Phi_{imj}^{g,w}$ act on it:

$$\begin{aligned} m^{w}\cdot\ddot{x}_{im}^{w} &= \sum_{j=1}^{2}\left(X_{imj}^{s,w}+X_{imj}^{r,w}\right) \\ m^{w}\cdot\ddot{y}_{im}^{w} &= \sum_{j=1}^{2}\left(Y_{imj}^{s,w}+Y_{imj}^{r,w}\right)-m^{w}\cdot a_{0} \\ m^{w}\cdot\ddot{z}_{im}^{w} &= \sum_{j=1}^{2}\left(Z_{imj}^{s,w}+Z_{imj}^{r,w}\right)+m^{w}\cdot g \\ I_{x}^{w}\cdot\ddot{\vartheta}_{im}^{w} &= \sum_{j=1}^{2}\left[\left(-1\right)^{j}\cdot b_{1}\cdot Z_{imj}^{s,w}+\left(-1\right)^{j}\cdot b\cdot Z_{imj}^{r,w}-r_{imj}\cdot Y_{imj}^{r,w}+\Theta_{imj}^{r,w}\right] \\ I_{y}^{w}\cdot\ddot{\varphi}_{im}^{w} &= \sum_{j=1}^{2}\left(r_{imj}\cdot X_{imj}^{r,w}+\Phi_{imj}^{r,w}-\Phi_{imj}^{g,w}\right) \\ I_{z}^{w}\cdot\ddot{\psi}_{im}^{w} &= \sum_{j=1}^{2}\left[\left(-1\right)^{j}\cdot b_{1}\cdot X_{imj}^{s,w}-\left(-1\right)^{j}\cdot b\cdot X_{imj}^{r,w}+\Psi_{imj}^{s,w}+\Psi_{imj}^{r,w}\right] \end{aligned}, \tag{4}$$

where $2\cdot b$ is a distance between wheel circles and r_{imj} is a radius of the corresponding wheel.

The mathematical model provides for the possibility of using PDS wheels (with independent rotation of the guide discs of the wheels relative to the corresponding supporting parts of the wheels around their common axis). Each guide disk of the PDS wheel (flange) has one degree of freedom. From the side of the rails, a moment acts on him $\Phi_{imj}^{r,g}$ and from the side of the corresponding supporting part of the wheel – the moment $\Phi_{imj}^{w,g}$.

Equations of motion of the guide discs of the PDS wheel:

$$I_{y}^{g}\cdot\ddot{\varphi}_{imj}^{g}=\Phi_{imj}^{w,g}+\Phi_{imj}^{r,g}, \tag{5}$$

where I_{y}^{g} is a moment of inertia of the guide disk of the PDS wheel relative to its axis of rotation and $\Phi_{imj}^{r,g}$ is a moment of creep forces acting on the *j*-th guide disk of the *i*-th wheelset of the *m*-th wagon bogie in the longitudinal vertical plane.

The moment $\Phi^{w,g}_{imj}$ is actually a moment of resistance in the junction of the support part of the wheel and the guide disk. The magnitude of this moment depends on the chosen design of the mating node and, in the general case, is determined by an expression of the form:

$$\Phi^{w,g}_{imj} = M_0 \cdot sign\left(\dot{\varphi}^{w}_{imj} - \dot{\varphi}^{g}_{imj}\right), \tag{6}$$

where M_0 is the maximum value of the moment of resistance at the interface of the supporting part of the wheel and the guide disk (depends on the design of the unit).

The expressions for determining the corresponding force factors in the equations are given below.

Refusal from commonly entered communication $\dot{\varphi}^{w}_{imj} = v / r_{imj}$ essential for studying the processes of resistance to movement and wear of the surfaces of the guide discs of the wheels, since in the curves of small radius, in which the highest intensity of these processes is observed, wheelsets move with significant slippage, and the above relationship becomes incorrect. In addition, such a model becomes suitable for studying transient modes of motion.

The rail vibration equations have the form:

$$\begin{aligned} m^r_g \cdot \ddot{y}^r_{imj} &= -k^t_g \cdot y^r_{imj} - \beta^t_g \cdot \dot{y}^r_{imj} + Y^{w,r}_{imj} \\ m^r_v \cdot \ddot{z}^r_{imj} &= -k^t_v \cdot z^r_{imj} - \beta^t_v \cdot \dot{z}^r_{imj} + Z^{w,r}_{imj} \end{aligned}, \tag{7}$$

where m^r_g is a reduced mass of the track in the horizontal direction, m^r_v is a reduced mass of the track in the vertical direction, k^t_g is a track stiffness coefficient in the horizontal direction, k^t_v track stiffness coefficient in the vertical direction, β^t_g is a horizontal viscosity coefficient, β^t_g is a path viscosity coefficient in the vertical direction.

Force interaction of the support part of the wheel and the corresponding guide disk in the direction of the coordinate φ_i was simulated by a constant moment of resistance $I'_{\tilde{n}}$ at their interface.

The results of numerous studies [10–14, 16] indicate that the tangential forces of interaction of deformable wheels and rails can be expressed in terms of the relative slip rates and loads at the points of contact. Modern models of tangential forces in wheel–rail contact (creep forces) are based on nonlinear dependencies of the form:

$$F_{x,y} = f\left(N, \varepsilon_x, \varepsilon_y, \phi, p\right), \tag{8}$$

where F_x, F_y are longitudinal and transverse creep forces acting tangentially to the point of contact of the plane, N is a normal reaction at the point of contact, ε_x, ε_y are longitudinal and transverse components of the relative slip at the point of contact, Φ is a spin and p is a set of geometric parameters characterizing wheel and rail profiles at the point of contact.

The components of the relative slip at the points of contact between the wheel and the rail, respectively, in the longitudinal ε_x and transverse ε_y directions and total relative slippage ε were calculated according to the following dependencies:

$$\varepsilon_x = \frac{V_{ck \cdot x}}{V}, \quad \varepsilon_y = \frac{V_{ck \cdot y}}{V}, \quad \varepsilon = \sqrt{\varepsilon_x^2 + \varepsilon_y^2}, \tag{9}$$

where $v_{ck\cdot x}$ and $v_{ck\cdot y}$ are the corresponding components of the speed of the wheel point at the point of contact with respect to the rail and *v* is the speed of the longitudinal movement of the wheelset.

In the compiled mathematical model, the calculation of the creep forces of each wheelset was carried out according to the following nonlinear analytical dependences with the setting of the creep coefficient k_c according to Müller [13, 16]:

$$\begin{gathered} k_c = P\cdot\left[235 - P\cdot\left(2.4 - 0.01\cdot P\right)\right] \\ P = 0.001\cdot N \\ F_{xy} = -\frac{1000\cdot k_c}{\left[1+\left(k_c\cdot\varepsilon/f/P\right)^m\right]^{\frac{1}{m}}},\quad F_x = -\varepsilon_x\cdot F_{xy},\quad F_y = -\varepsilon_y\cdot F_{xy} \end{gathered}, \tag{10}$$

where F_{xy}, F_x and F_y are the force of the creep at the point of contact and its longitudinal and transverse components, respectively and *f* is a friction coefficient. Recommended values of parameter m = 3, 4 [16].

The relative slip at the contact points was determined as follows:

- in the main contact of the wheel with the rail,

$$\varepsilon_{yi} = \frac{1}{v}\cdot\left[\left(\dot{y}-\dot{y}_{Pi}\right) - v\cdot\psi - r_i\cdot\dot{\varphi}_i\cdot\psi\right], \tag{11}$$

$$\varepsilon_{xi} = \frac{1}{v}\cdot\left[\left(-1\right)^{i-1}\cdot S\cdot\dot{\psi} + \left(-1\right)^{i-1}\cdot\frac{v\cdot\left(y-y_{Pi}\right)}{r_i} + \left(-1\right)^{i-1}\cdot\dot{\varphi}_i\cdot\lambda\cdot\left(y-y_{Pi}\right) + r_i\cdot\dot{\varphi}_i\right], \tag{12}$$

where r_i is a current rolling radius of the left (i = 1) and right (i = 2) wheels:

$$r_i = r + \left(-1\right)^i\cdot y\cdot\lambda, \tag{13}$$

- in flange contact:

$$\varepsilon^f_{xi} = \frac{1}{v}\cdot\left[v + \left(-1\right)^{i-1}\cdot S\cdot\dot{\psi} - r_{fi}\cdot\dot{\varphi}_{fi}\right], \tag{14}$$

$$\varepsilon^f_{yi} = \frac{1}{v}\cdot\left[\left(\dot{y}-\dot{y}_{pi}\right) - v\cdot\psi - r_{fi}\cdot\dot{\varphi}_{fi}\cdot\psi\right], \tag{15}$$

where r_{fi} is a distance from the point of the flange contact to the axis of rotation of the guide disc (flange).

The direction of the vectors of slippage velocities and creep forces in the flange contacts were determined in accordance with the established features of the movement of the PDS wheels [1, 2].

Coordinates φ_1 and φ_2 were taken as small angles of rotation of the wheels relative to the common axis of the wheelset, taking into account its final torsional stiffness c_0.

The comparison of resistance indicators to the movement of cars with different wheel designs was made on the basis of a comparison of several indicators, the main of which was the total specific resistance to the movement of the wagon w''_u [N/kN].

The value of this indicator was determined on the basis of data sources [20, 21], according to which the total drag of the wagon:

$$w''_u = w''_{um} + w''_{ua}, \tag{16}$$

where w''_{um} is the main resistivity of the wagon and w''_{ua} is the additional resistivity of the wagon.

Among the known works to determine the components of the main resistivity of rail crews are studies [20], in which it is noted that the main components of this resistance are the resistivity from friction in the bearings w''_b, the resistivity of the rolling friction of the wheels on the rails w''_{rf}, the resistivity of the sliding friction of the wheels on the rails w''_{sf}, aerodynamic drag w''_{ad}, resistivity from energy dissipation in the track w''_{rt} and resistivity from energy dissipation in the environment w''_e. The values of w''_b, w''_{rf}, w''_{ad} and w''_{rt} were calculated according to the recommendations [20, 21].

Thus, the value of resistivity from friction in bearings is determined from the expression:

$$w''_b = \frac{150 \cdot \left(q_0 - 1.2\right) \cdot \mu}{q_0}, \tag{17}$$

where q_0 is the axial load of the wagon and μ is a value of the coefficient of friction of roller bearings depending on the speed and load on the axle.

The value of the rolling resistance from wheel friction on rails w''_{rf} is difficult to quantify using generalized empirical formulas, because under the influence of large loads and plastic deformations of the material simultaneously with rolling friction microprocesses of sliding friction and wear of contact surfaces. Therefore, the value of this value according to the recommendations defined in the work [20], in the calculations was taken in the range:

$$w''_{rf} = 0.3 \div 0.4 \left[\mathrm{N/kN}\right], \tag{18}$$

Specific aerodynamic drag w''_{ad} of a wagon movement first of all depends on the factors connected with speed of movement, axial loading and ambient temperature:

$$w''_{ad} = \frac{0.69 \cdot v^2}{q_0 \cdot T}, \tag{19}$$

where v is a wagon speed, q_0 is the axial load of the car and T is air temperature [K]. It is known that the condition of the track significantly affects the overall resistance of the crew, especially with increasing speeds and axial loads. The expression for determining the resistivity of the wagon from energy dissipation in the track was used in the form [20]:

$$w''_{rt} = 0.000035 \cdot v^2 + 0.0036 \cdot q_0, \tag{20}$$

The value of the resistivity from energy dissipation into the environment w''_e is also difficult to quantify using generalized empirical formulas due to the presence of a large number of factors that affect its magnitude. In comparative calculations, the values of the value of w''_e were taken according to [20] for a standard three-element carriage with unworn elements of the spring suspension.

The value of w''_{sf} of the resistance of the car of sliding friction of the wheels on the rails was determined by converting the specific values [N/kN] of the absolute values [N] of the forces of resistance to sliding of the wheels of the car w_{sf} , obtained by calculating the above mathematical model, i.e.:

$$w''_{sf} = \frac{w_{sf}}{q_0}, \tag{21}$$

The additional resistivity of the wagon w''_{ua} in the general case includes the resistivity from the curve w''_{uar} , the resistivity from the slope of the track w''_{uai} and the resistivity from the wind load w''_{uaw} . It should be noted that the values of the last two components cannot be influenced by using the proposed option to improve the design of the wheel. Since the aim of the study is to compare the values of drag of the car crews with the wheels of TDS (traditional design scheme) and PDS, assuming the same conditions of movement of the compared crews, the values of these components of the additional resistivity of the car in comparative calculations were taken as nought.

To calculate the specific additional resistance, only the specific resistance to motion in the curve w''_{uar} was taken into account. The calculation of the value of this resistance was also carried out in accordance with the provisions of [20]:

$$w''_{uar} = \frac{100}{R} + 1.5 \cdot |\tau|, \tag{22}$$

where *R* is a radius of the curve [m] and |*τ*| the absolute value of the outstanding transverse acceleration in the curve [m/s^2].

4 RESULTS OF THE RESEACH AND DISCUSSION

To integrate the system of equations, a two-step Runge-Kutta method of the second order was used [16], the application of which is due to the presence in the simulated system of a large number of high stiffness springs, one-sided limiters with gaps and dry friction elements that make part of the equation system smooth. The integration step was selected experimentally and was usually taken equal to several hundred thousandths of a second.

On the basis of the compiled mathematical model, software for a computer was developed, which implements the integration of the above equations of motion of the crew, and preliminary calculations were carried out.

In Fig. 2 shows the results of a comparison of some calculated dynamic indicators with experimental data, respectively, for the empty (a) and loaded (b) state of the car for the case of its movement in a straight section of track. The indicators are the vertical $\ddot{z}$ and horizontal $\ddot{y}$ accelerations of the body (in fractions of *g*) in the centre of gravity, the coefficients of the vertical body dynamics K_{vd} and maximum ratios of horizontal forces in the flange contact to the static load *Y/G* from the wheel to the rail [22, 23].

The circles show the values of the corresponding indicators obtained as a result of field tests, solid lines – according to the results of calculations on a mathematical model. The horizontal dashed lines show the permissible values of the corresponding dynamic indicators [24–25].

The values of the dynamic indicators of gondola cars obtained during field tests on the 1520 mm gauge railways of Ukraine and the Russian Federation available from literature sources [17–19], were taken as experimental.

The quantitative agreement between the calculated and experimental values of the quantities under consideration is satisfactory. The maximum discrepancy between the results does not exceed 9% for a loaded car and 16% for an empty car. This confirmed the suitability of the compiled mathematical model and software for solving the problem.

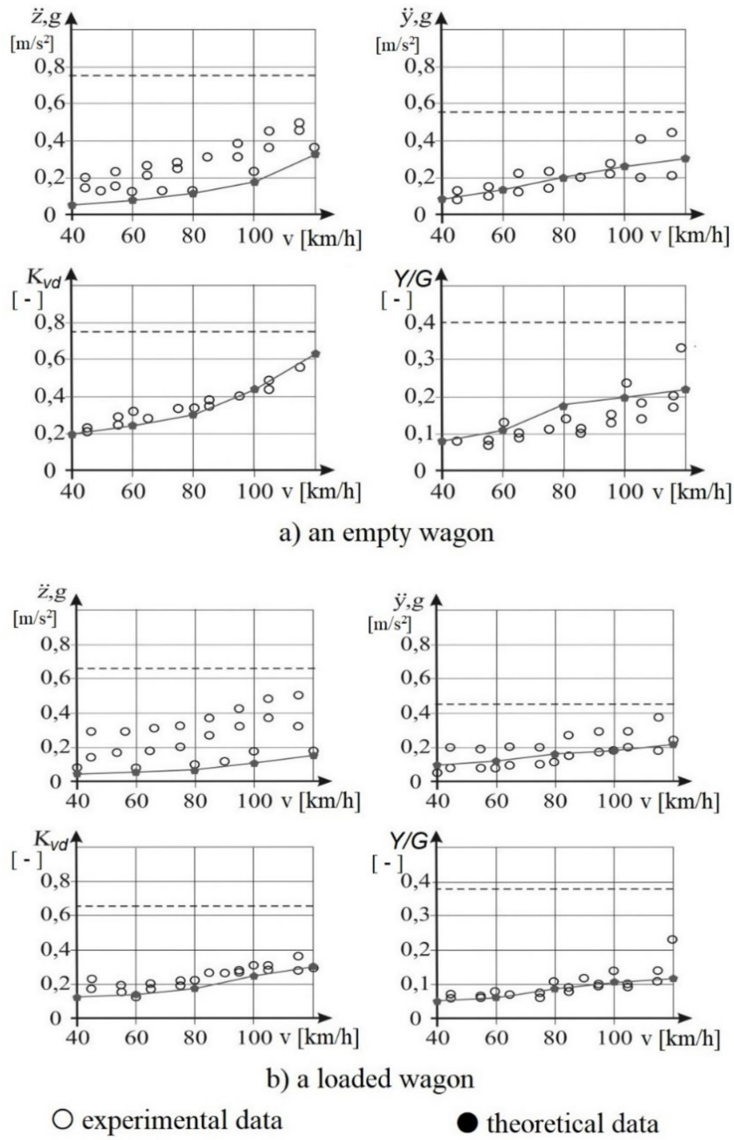

Fig. 2 Dynamic performance comparison results

On the described mathematical model, comparative calculations of indicators of dynamics and resistance to movement of cars with wheels of different design schemes are carried out. The analysis of the calculation results showed that the change in the design of the wheel had practically no effect on the dynamic performance of the car movement. Some results of calculations concerning the resistance to the movement of cars are presented below.

The calculated dependences on the speed of movement of the total resistivity to the movement of an empty and loaded car with wheels of different design schemes are shown in Fig. 3 to Fig. 6.

The analysis of the obtained results showed a rather close coincidence of the nature of the calculated curves with the empirical curves, which were constructed on the basis of the formulas of the normative document – the Traction Calculation Rules (TCR) for train operation [21]. The quantitative agreement between the calculated and experimental values of the considered quantities is also satisfactory. The maximum discrepancy between the results does not exceed 11% for a loaded car and 17% for an empty car.

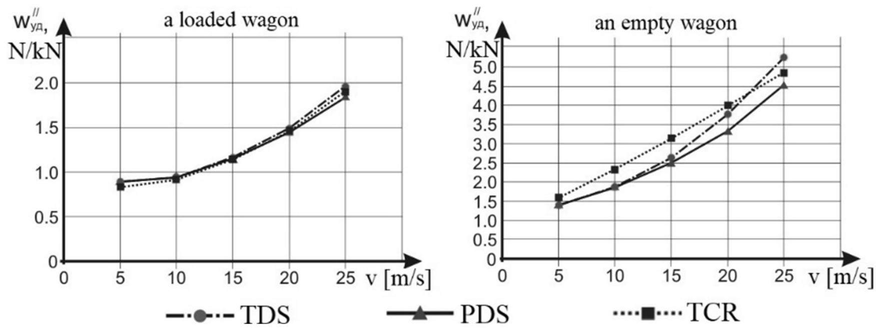

Fig. 3 Total resistivity to the movement of the car in the straight section of the track

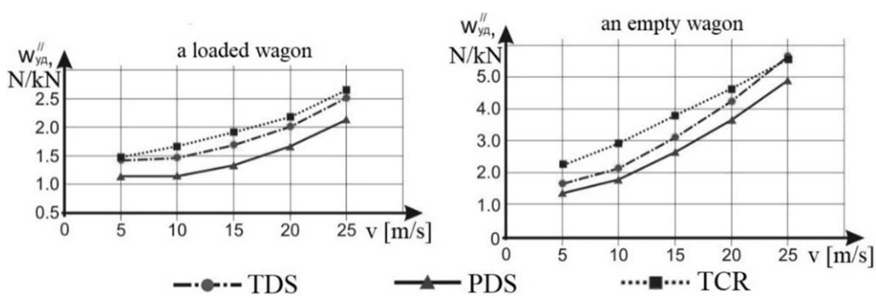

Fig. 4 Total resistivity to the movement of the car in the curve R = 1200 m

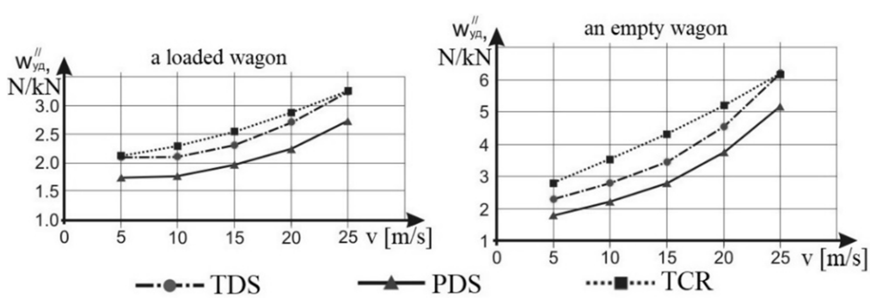

Fig. 5 Total resistivity to the movement of the car in the curve R = 750 m

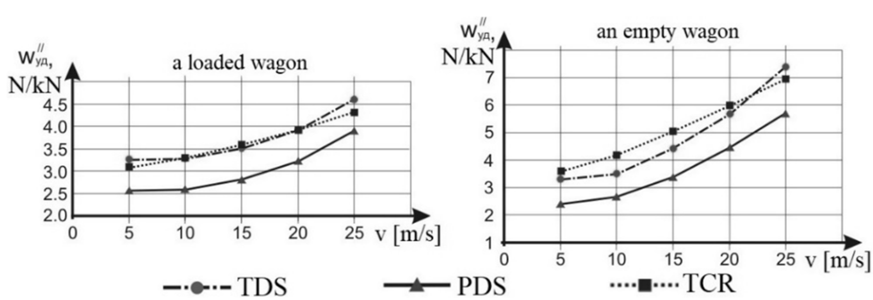

Fig. 6 Total resistivity to the movement of the car in the curve R = 350 m

Based on the analysis of the plotted graphs, it was found that the values of the total resistivity during the movement of an empty and loaded car with PDS wheels in various modes of movement are lower than the values of this indicator with TDS wheels under the same conditions.

In straight sections of the track, this reduction can be, depending on the speed of movement, 10 ... 12% for a loaded car and 13 ... 15% for an empty car. In curves with a radius of 350 m, 750 m and 1200 m, the decrease can be, respectively, 20 ... 22% and 23 ... 25%, 16 ... 18% and 17 ... 19%, 13 ... 15% and 14 ... 16%.

5 CONCLUSION

Analysis of the results of mathematical modelling of the movement of a four-axle freight car with wheels of different design schemes along a rail track indicates that the use of wheels of a promising design scheme in the undercarriage of rail vehicles can reduce the resistance to the movement of rail rolling stock by minimizing the kinematic slipping of the wheel flanges on the rails. This confirms the expediency of using such wheels in the carriage parts of cars to reduce the cost of traction for trains and reduce wear of the contacting surfaces of the wheel flanges and side faces of the rail heads.

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